October 2009

Voting, the Symmetric Group, and Representation Theory
By: Zajj Daugherty, Alexander K. Eustis, Gregory Minton, and Michael E. Orrison
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We show how voting may be viewed naturally from an algebraic perspective by viewing voting profiles as elements of certain well-studied $\mathbb{Q}S_n$-modules. By using only a handful of simple combinatorial objects (e.g., tabloids) and some basic ideas from representation theory (e.g., Schur's Lemma), this allows us to recast and extend some well-known results in the field of voting theory.

A Bijective Proof for a Theorem of Ehrhart
By: Steven V Sam
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Given a lattice polytope, a theorem of E. Ehrhart states that the number of lattice points in its integral dilates is counted by a polynomial. A surprising reciprocity law says that the number of interior lattice points of these dilates is obtained by substituting negative values into this polynomial. The usual combinatorial proof of this theorem is via generating functions, but we offer a more geometric proof via recurrence relations and inclusion-exclusion. This perspective shows that reciprocity is actually quite natural. We conclude with further directions for the interested reader.

Sudoku: Strategy Versus Structure
By: J. Scott Provan
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Sudoku puzzles, and their variants, have become extremely popular in the last decade. They can now be found in major U.S. newspapers, puzzle books, and web sites; almost as pervasive are the many guides to Sudoku strategy and logic. We give a class of solution strategies--encompassing a dozen or so differently named solution rules found in these guides--that is at once simple, popular, and powerful. We then show the relationship of this class to the modeling of Sudoku puzzles as assignment problems and as unique nonnegative solutions to linear equations. The results provide excellent applications of principles commonly presented in introductory classes in finite mathematics and combinatorial optimization, and point as well to some interesting open research problems in the area.

Partial Metric Spaces
By: Michael Bukatin, Ralph Kopperman, Steve Matthews, and Homeira Pajoohesh
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When mathematics is processed on a computer, objects are known only to the extent to which their
values are computed; the metric space axiom that says \( d(x,x)=0 \) for each point \( x \) then becomes the unrealistic assumption that we always know the eventual value of \( x \) exactly.

The theory of partial metric spaces generalizes that of metric spaces by dropping that axiom to allow structures that simultaneously model mathematics and its computer representation. In them, \( d(x,x)=0 \) for the ideal, completely known points; \( d(x,x)\neq0 \) for their partially computed approximations. We discuss how familiar metric and topological reasoning is refined to work in the general setting of convergence and continuity which can now be represented on computers.

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**The Sixty-Ninth William Lowell Putnam Mathematical Competition**
By: Leonard F. Klosinski, Gerald L. Alexanderson, and Loren C. Larson

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**Notes**

**Why Is \( PSL(2,7) \cong GL(3,2) \)?**
By: Ezra Brown and Nicholas Loehr
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We construct a straightforward, explicit isomorphism from the group \( PSL(2,7) \) onto the group \( GL(3,2) \). Our proof requires only elementary facts about groups, fields, and matrices. In particular, our proof does not utilize projective geometry, block designs, or simplicity arguments.

**Angles as Probabilities**
By: David V. Feldman and Daniel A. Klain
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Almost everyone knows that the inner angles of a triangle sum to 180 degrees. But what happens if you sum the solid inner angles over the vertices of a tetrahedron \( T \)? Divide this sum by \( 2\pi \), and you have the probability that the orthogonal projection of \( T \) onto a random 2-plane is a triangle. We prove this, along with a more general theorem for simplices in \( n \)-dimensional Euclidean space.

**Jump Home and Shift: An Acyclic Operation on Permutations**
By: Villo Csiszár
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Suppose you have a number of files on a shelf, which should be in a prescribed order, but are mixed up. Take one out, put it in its proper place, and shift the other files as necessary. We show that however clumsily you keep doing this, the files will eventually be sorted, i.e., you cannot get into an infinite cycle. We also describe the connection of this jump-home-and-shift operation to a containment relation on permutations defined by P. McCullagh, and thus prove a conjecture of his.

**Ptolemy Meets Erdős and Mordell Again**
By: Hojoo Lee
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We apply Ptolemy's theorem to obtain refinements of the weighted Erdős-Mordell theorem.

**On the Irreducibility of Polynomials with Leading Coefficient Divisible by a Large Prime Power**
By: Anca I. Bonciocat and Nicolae C. Bonciocat
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Integer polynomials with some prescribed nonzero coefficients are irreducible if their leading coefficients are divisible by a large enough prime power. In this paper, we provide an explicit condition on the size of such a prime power that ensures the irreducibility of polynomials of this type.
Reviews

*Roots to Research: A Vertical Development of Mathematical Problems*

By: Judith D. Sally and Paul J. Sally, Jr.
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